

# The Persistence and Predictability of Closed-End Fund Discounts

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# The Closed-End Fund Discount Persistence and Predictability

## Abstract

It is well-known that the level of closed-end fund discounts appears to predict the corresponding fund's future returns. We further document that such predictability decays slowly. The popular explanations, including the tax effect, investor sentiment risk, and the funds's dividend yield, do not fully account for the observed predictability. At the same time, discounts are very persistent especially on an aggregate level. Using an AR(1) model for discounts, we demonstrate that such predictability is largely due to persistence in discounts. Our calibration exercise can produce most characteristics of an aggregate equity close-end fund index over the ten year period from 1993 to 2001. A cross-sectional study links discount persistence to rational factors such as expense ratios, dividend yield, unrealized capital gains, and turnover. In addition, we document a second independent source for predicting fund returns from large stock portfolio returns. This evidence suggests that the well-known lead lag relationship between large stocks and small stocks also exists between NAV returns and fund returns. Finally, we find no evidence for "excess volatility" on the aggregate level both for conditional and unconditional volatility.

**Key words:** closed-end fund, cross-correlation, discount, excess volatility, investor sentiment, large stocks, persistence, small stocks, turnover

# Introduction

One of the most enduring conundrums in the field of finance is generally described as “the closed-end fund” puzzle. Unlike a regular (open-end) mutual fund, which sells new shares at net asset value (in some cases plus a sales charge) and also redeems shares at the net asset value (in some cases minus a redemption fee), a closed-end fund issues a fixed number of shares that then trade in the stock market just like an ordinary stock. Holders of shares who wish to liquidate must sell their shares to other investors. The shares are typically issued at net asset value (NAV) plus a fee to defray underwriting costs. Thus, the fund begins life selling at a premium. But typically, within months, the stock of the fund persistently sells at a discount to NAV. This persistent discount appears to violate the law of one price and constitutes the closed-end fund puzzle.

If we believe that the stock market is largely efficient, the assets held by a closed-end fund should be priced correctly. The closed-end fund investors should also price these assets in the portfolio that is the fund, in the same way. In other words, if both investors in the underlying assets and investors in the closed-end fund are similar, and there are no market imperfections or market frictions, there should not be a discount. The literature on discounts explores either irrational explanations based on investor sentiment and market imperfections, or rational causes such as expenses, taxes, etc. We shall see that the implications of the two alternative sets of explanations lead to different empirical expectations. Irrational explanations tend to be favored by the behavioral school of finance. Rational ones are likely to be favored by finance scholars who believe that markets are generally quite efficient.

## Behavioral and Imperfect Market Explanations of Closed-end Fund Discounts

1. Investor Sentiment and Noise Trading (DeLong et al., 1990, Lee, Shleifer, and Thaler, 1991)

The purchase of a closed-end fund takes on two kinds of risks: First there is the risk of fluctuations in the underlying assets held by the fund. Additionally, the fund owner bears the risk that changes in market sentiment will cause fluctuations in the demand for closed-end funds. If the market is imperfect in

exploiting the potential arbitrage opportunities, the closed-end fund will sell at a discount to compensate investors for this added risk. Discounts are also likely to fluctuate with sentiment risk. The empirical implication of this hypothesis is that the volatility of the fund shares will be greater than the volatility of the funds's assets. Moreover, the sentiments of noise traders may not revert to the mean for long periods of time and discounts could even become more extreme in the future.

2. Costly Arbitrage (Pontiff, 1996; Gemmill and Thomas, 2002)

One might think that arbitrage would bring the price of the fund and net asset value together. But arbitrage may be costly and it may be difficult to replicate the fund's portfolio, especially since the exact composition of the portfolio cannot be known for certain and many of the securities in the portfolio may be illiquid. Therefore, if irrational forces in the market create difference between fund prices and the corresponding net asset values, the difference is likely to persist. The argument has an indirect empirical implication on the effect of the dividend yield on fund discounts. The party that holds a short position is obligated to pay dividends. Arbitrage is easier if the dividend yield on the short position (the securities in the fund that are shorted) is less than the dividend yield on the long position held in fund shares, i.e., the dividend yield on the purchase of fund shares is greater than the dividend that must be paid by the arbitrageur who is short the fund's assets. Thus, funds with higher dividend yields will tend to have lower discounts. Moreover, the less liquid are the fund's securities, the greater are the transactions costs involved and the costs for the arbitrage transaction. Pontiff (1996) uses residual variance as a proxy for the ability to arbitrage. If a closed-end fund's return is very close to the market return, that is residual variance is small, it is easy to arbitrage regardless of the knowledge of underlying portfolio.

3. Excess Volatility of Fund Share Prices (Pontiff, 1997)

If investor sentiment leads to varying discounts then the variance of closed-end fund returns will exceed the variance in the fund's NAV. When we consider the NAV as the ex post value while the fund price as the ex ante value, this phenomenon is similar to the finding of "excess volatility" of Shiller (1981). One

caveat to this statement is that high volatility in fund return occurs only when the innovation in the discount is uncorrelated with the underlying asset returns. As we will discuss further, this is not the case in our sample.

### Rational Explanations of Closed-end Fund Discounts

#### 1. Expense Ratio (Ross, 2004)

A closed end fund manager benefits from collecting periodic management fees, while fund investors are paid with periodic dividends. Assume that  $e$  is the percentage of the fund's NAV paid out as an expense ratio (management fee), and  $\delta$  is the percentage of its NAV paid as a dividend (the dividend yield). Clearly, the fund investors only have portion of the claim for the fund's future cash flows. Therefore, a fund investor could only be willing to pay for the fund an  $\frac{\delta}{\delta+e}$  percentage of the NAV, or a discount of  $\frac{\delta}{\delta+e}$ .

#### 2. Tax Considerations (Malkiel, 1977, 1995)

Whenever a closed-end fund sells a security at a gain, it generates a taxable event. These gains are passed to investors who will pay capital gains taxes. This inability to postpone the realization of capital gains has negative consequences. Funds with large percentages of unrealized capital appreciation can be liquidated after tax at only a fraction of the fund's NAV.

#### 3. Dividend Yield (Pontiff 1996)

A higher dividend yield on the fund makes arbitrage less costly since it is easier to cover the dividend obligation on the short position in the underlying assets. Therefore, one should expect to see a negative relationship between dividend yield and discounts. Also, a higher dividend yield may be particularly beneficial to small investors who are living on the income from their investments. Dividends can be spent with no transactions costs. Liquidation of shares for living expenses involves transactions costs.

#### 4. Liquidity of Fund Holdings

Liquidity can have both positive and negative impact on closed-end fund prices. On the one hand, illiquid holdings may be difficult to value (Malkiel 1997) and will make arbitrage more costly (Gemmill and Thomas, 2002) thus leading to

larger discounts. On the other hand, there are clientele effects. The closed-end form may be best suited to illiquid investments and will give small investors an effective means to hold the securities, for example single state municipal bonds. This efficiency gain would lead to smaller discounts (See Cherkes, 2003).

Whether behavioral or rational arguments are more important in affecting closed-end fund discounts is an empirical question. As Malkiel (1997) has shown, the rational explanations can account for at least some portion of the cross-sectional differences in discounts. It is important, however, to further investigate their empirical implications in a time series study, especially their impact on the dynamic properties of discounts. First, expense ratios, tax penalties, dividend yields, liquidity of assets are all relatively stable. In particular, any change in expense ratio requires shareholders' approval. At the same time, closed-end funds seems to follow a relatively stable distribution policy in realizing capital gains, which implies a relatively stable dividend yield. Moreover, a closed-end fund that holds illiquid assets follows a stable long-term investment objective. Therefore, these rational arguments imply that discounts will be relatively stable over time. More importantly, a shock to discounts will have a prolonged impact if discounts are indeed related to these factors. In other words, discounts will be persistent. Second, to a large extent, the rational explanations imply that fund prices should not display excess volatility. Finally, fund returns should then be somewhat predictable.

For ease of discussion, suppose both fund price and NAV are all constant, that there are no capital gains, and that discounts are constant over time. Since returns are only generated from dividend yields earned on the NAV, the larger the discount, the lower the fund price, and thus the higher the future fund return.<sup>1</sup> This suggests that if expense ratios are similar for all funds, a large current discount predicts a high future fund return. Such a predictability is generated without any predictability in the underlying assets. This analysis relies only on the constant discount assumption. Of course, the main determinant of the fund return will be the investment results, i.e. the behavior of the NAV over time. Thus, we should expect only a small prediction

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<sup>1</sup>Moreover, to the extent that discounts result from the fund's holding of illiquid assets, these holdings ought to be priced to generate higher returns on the NAV.

effect from the discount (and its persistence). We will fully develop this idea in the next section.

Although there is a large literature in understanding closed-end fund discounts, limited attention has been focused on the dynamics of discounts and fund returns. The most comprehensive study on this issue is by Pontiff (1995). He tries to explain the positive correlation between current discounts and future fund return first documented by Thompson (1978). The three possible explanations used are investor sentiment risk, bid and asked spread, and the dividend effect. His empirical findings do not seem to support these explanations, however.

In this paper we take a different approach. Drawing from the rich literature on closed-end fund discounts, we ask what is the implication of persistent discounts on the dynamics of fund returns. As we will show that discounts and fund returns are related, it is difficult to find a separate set of explanation for the predictability of returns. Starting from a hypothesis of a persistent discount process backed by existing explanations for discounts, we focus on the rich dynamics of aggregate discounts and aggregate returns. Modeling persistence is a useful strategy. For example, Stock and Watson (1991) model a persistent factor that determines the business cycle. Conrad and Kaul (1988) model the expected return to be persistent. More recently, Bansal and Yaron (2004) are able to explain the equity premium by modeling both consumption growth and dividend growth as containing a persistent factor. In most of these studies, the persistent factor is unobservable. In the current study, discounts are persistent and are readily observable. Indeed, using aggregate equity closed-end fund data, we find that 1) discounts are very persistent with an average level of 8%; 2) although returns from the underlying assets are close to i.e., we shall see that these NAV returns predict future fund returns; 3) discounts at “any” lags also predict future fund returns; and 4) there is no “excess volatility” in our sample. Moreover, our model is capable in generating these stylized facts.

Also related to this paper is a recent study by Day, Li, and Xu (2005). They investigate the predictability of discounts on an individual fund level using an adaptive expectations model. In particular, investors adjust their expectation about future fund returns according to innovations in NAV returns. Their model not only generates

predictability of NAV returns for future fund returns, but also links current changes in discounts to future fund returns. In contrast, we study the predictability on the aggregate level. As a natural extension, we are able to link our findings to Swami Nathan's (1996) study that finds that fund discounts also predict future returns of small stock, but not large stocks. What is even more interesting, we find that it is the fund returns, rather than discounts, that have most predictability power for future small stock returns. This result can be understood if both closed-end fund underlying assets and small stock portfolios are subject to the same liquidity shock.

Finally, we document a second independent source for predicting fund returns from large stock portfolio returns. Lo, and MacKinlay (1990) have shown that large stock portfolios lead small stock portfolios but not the other way around. Since closed-end fund investors consist largely of small investors, their trading behavior might resemble those of small stock investors. Therefore, we may not only expect to see the lead lag relationship between NAV returns and fund returns, but also that large stock portfolios might predict fund returns. This is exactly what we find in our sample.

The paper is organized as follows. We discuss the characteristics of our sample and the construction of a closed-end fund index in the next section. We also reexamine the useful rational factors in explaining both the level and the persistence in discounts in the same section. In section 2, we study how fund discounts and returns of large stocks have some predictive power for future fund returns. Related to the dynamics of closed-end funds is the volatility issue. We investigate "excess volatility" on individual fund level and the conditional volatilities for both fund returns and NAV returns on aggregate level in section 3. Concluding comments are offered in section 4.

# 1 The Characteristics of Closed-end Fund Returns and Factors that Determine Discounts

Most closed-end fund studies focus on explaining the existence of fund discounts. This paper investigates the dynamic properties of aggregate equity closed-end fund discounts. We obtained information on 59 equity closed-end funds from Morningstar for a ten year period from 1993-2002.<sup>2</sup> The data set includes weekly fund prices and NAV values. Since closed-end funds exhibit very different behavior during both the IPO period and the open-ending period, during which funds may convert to open-end funds, we delete the first six month data if a fund had an initial public offering or the last six month data if a fund open-ended during our sample. In addition, we required a fund to have at least three years of data.<sup>3</sup> As a convention, we aggregated individual fund information including fund returns, NAV returns, and discounts using equal weights. We believe that equal weighting is representative of the characteristics of individual funds, and it is the most widely used approach in the literature

## 1.1 The Characteristics of the Closed-end Fund Sample

We first provide summary statistics for aggregate fund returns, NAV returns and discounts in Table 1. The average discount of 8.42% is surprisingly close to that documented in Lee, Shleifer, and Thaler (1991) for an earlier sample period. Despite the high persistence in the aggregate discount (with an auto correlation of 98%), discounts had a large swing over our sample period with a standard deviation of 3.62%. The average weekly fund return and NAV return are both about 0.145% compared to an average market return of .187% for the same period. Nevertheless,

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<sup>2</sup>We are grateful to Morningstar for making the data available to us.

<sup>3</sup>We made two technical adjustments. There was a huge drop in the week of September 14, 2001 and a reversal of almost the same amount in the following week for most funds. This irregular movement caused two large outliers in the aggregate data. Therefore, we smoothed out these two periods. In addition, fund “Engex Inc.” had a tremendous jump in its share price at the beginning of 2000 from \$20 to \$50, then returned to it’s the original level. We also smooth this irregular period for this fund to ensure that errors in our data set do not influence the results.

the Sharpe ratio for closed-end fund series was 0.1107, which is much higher than that the market Sharpe ratio of 0.0806. It is interesting to note that fund returns exhibit large positive autocorrelation (22.59%) while NAV returns have virtually no autocorrelation.<sup>4</sup> As a comparison, the market return over the same period has a small negative autocorrelation of  $-7.31\%$ .

Insert Table 1 Approximately Here

In contrast to Pontiff (1998), we do not see “excess volatility” in the aggregate fund returns.<sup>5</sup> In fact, the volatility for the fund return is only 1.31% while that of the NAV return is 1.42%. If it is difficult or impossible to arbitrage the underlying assets, this result is completely rational. We could interpret the NAV as the realized value while the fund price could as reflecting an expectation of future value. In the next section, we will discuss under what scenarios this interpretation is reasonable. Here, we offer an intuitive explanation to understand this phenomenon. In the last column of Table 1, we run the market model for both the aggregate NAV returns and the fund returns. Beta for the NAV returns (0.558) is much larger than that for the fund returns (0.427). This suggests that there are other “systematic” factors that are negatively correlated with the market factor. It is exactly this negative correlation from these closed-end fund factors that reduce the overall volatility of the fund returns. In fact, after controlling for “market” volatility, we do observe “excess volatility” in the residual returns with 0.86% for fund returns versus 0.58% for NAV returns.

Perhaps the most interesting dynamic feature about closed-end funds is the fact that discounts appear to (weakly) predict future fund returns as first documented by Thompson (1978). This weak predictability is confirmed in our sample. Results from univariate regressions in Table 1 suggests that the current discount predicts the future fund return with an  $R^2$  of 2.45%. Recently, Day, Li, and Xu (2005) suggested that the change in the discount also predicts future returns on the individual funds’ level. This predictability does not show up in the aggregate level (not shown in the

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<sup>4</sup>The confidence interval for the autocorrelation coefficient is  $1/\sqrt{520}$ .

<sup>5</sup>This conclusion does not depend on the particular weighting scheme used.

table). Since discounts are related to fund prices and NAVs, it is useful to study the dynamics of fund returns and NAV returns in order to understand this phenomenon. First, the underlying asset market seems to be “weak form” efficient, in the language of Fama, since current NAV returns do not depend on either past fund returns ( $r_{p,t}$ ), or past NAV returns ( $r_{v,t}$ ), although it is somewhat predictable by past discounts. When all of the three lagged variables are used in the following multiple regression,

$$r_{v,t+1} = -0.160 - 0.089r_{v,t} + 0.149r_{p,t} + 0.034d_t + e_t, \quad R^2 = 1.0\%, \quad (1)$$

(.158)   (.089)   (.098)   (.0173)

none of the variables are significant at a 5% level. This evidence supports the use of a random walk model for the NAV process in the next section.

In contrast, it is interesting to see that the current NAV return predicts the future fund return with an  $R^2$  of 5.92%. As suggested by Day, Li, and Xu (2005), this correlation could be attributed to time-varying adaptive expectations, where fund investors adjust their expectations about the fund’s future return according to shocks to the current NAV return. This correlation is largely responsible for the predictive power of changes in discounts on individual fund level. However, we will show in the next section, that it is not the main attribute of the predictability we find in the aggregate discount. In addition, we also observe sizable autocorrelation in fund returns. In particular, the  $R^2$  for an AR(1) model applied to aggregate fund returns is 5.11%. This could be attributed to the predictive power of discounts on fund returns.

The dynamic properties of discounts are influenced by other factors. It is interesting to see from the univariate regressions in Table 1 that the NAV return predicts future discounts with an  $R^2$  close to 1%. Since discounts are persistent while NAV returns are non-predictable, we should control for discount persistency. In other words, it might be the shocks to NAV returns that predict innovations in discounts ( $d_t$ ). This is confirmed in the following regression,

$$d_{t+1} = 0.109 + 0.990d_t - 0.169r_{v,t} + e_t \quad R^2 = 96.5\% \quad (2)$$

(.075)   (.008)   (.021)

Here we see that a positive NAV return in period  $t$  is associated with a reduction in the funds’ discount in period  $t + 1$ . One explanation for this negative correlation

between current NAV return and future discount innovation is that current NAV returns are positively correlated with future fund returns. By definition, future fund returns will influence the innovation in the future discounts. This is in the spirit of our model discussed in the next section. We will also use this estimate to calibrate our model.

## 1.2 What Might Determine Discount Level and Its Persistence?

As discussed in the introduction, many rational explanations for the existence of discounts are consistent with discount persistency. For example, Pontiff (1996) predicts that a higher dividend yield will be associated with low discounts. This is because a high dividend yield on the fund shares makes it easier to cover the dividend obligation in the short position of the underlying assets. In other words, arbitrage is less costly and reduce its duration. In addition, a higher dividend yield will benefit the small investors that use the income from their investments for living expenses. To test this hypothesis, we run cross-sectional regressions between individual funds' average discounts and dividend yields each year from 1999 to 2002.<sup>6</sup> The results are reported in Table 2

Insert Table 2 Approximately Here

The relationship between dividend yield and discounts is indeed negative as shown in the first row block of Table 2. The relationship is significant, however, in only two of the four years. At the same time, discount persistence seems to have a very significant positive relationship with dividend yield during any given year. This makes perfect sense. Recognizing many small investors are dependent on dividend income, fund managers tend to maintain a stable dividend policy. Therefore, any changes in the dividend policy will likely have permanent effects, which should make discounts persistent. Moreover, funds with large dividend yields tend to attract a clientele of

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<sup>6</sup>The characteristic information of individual funds are from Morningstar Closed-end Fund Disc, which is only available to us for the last four years.

investors with large investment income needs. Thus, changes in dividends will tend to have a prolonged impact, and discounts are likely to be persistent.

Taxes could also play a role in influencing the discounts on closed-end funds. As Malkiel (1977) has pointed out, even though the unrealized capital gains might not have been accumulated during the holding period of the current shareholders, when managers decide to realize these gains for reasons such as portfolio rebalancing, the realized gains are passed to investors who are then liable for capital gains taxes. Therefore, funds with larger percentages of unrealized capital appreciation should trade at larger discounts. The relationship is positive, although it is significant at a 5% level only for the last two years and significant at a 10% level for the first year as shown in the second left block of Table 2. This could be due to a second effect. High unrealized capital gains might signal that the fund has a conservative policy in realizing the capital gain. A policy of postponing the realization of capital gains has positive consequences, which should lower the discount. At the same time, we see that discount persistence is negatively related to the percentage of unrealized capital appreciation. Given that discounts are positively related to unrealized capital gains, any shock to discounts is more likely to come from a fund with a larger level of unrealized capital gains than a fund with a smaller level of unrealized capital gains, other things being equal. Since a fund with a large transitory component tends to have a smaller permanent component, discounts of a fund with large unrealized capital gains will tend to be less persistent.

Another factor that works through tax impact is turnover. When turnover is higher, capital gains will be realized more often. It is unlikely, therefore, that the fund will accumulate large unrealized capital gains. Therefore, turnover should have exactly the opposite impact on discounts or persistence compared to unrealized capital gains. From the third row block, we do observe a negative relationship between discounts and turnover while a positive relationship between persistence of discount and turnover. Of course, this variable is highly correlated with the above unrealized capital gain variable.

The managers of closed-end funds collect management fees throughout the year. Therefore, fund investors will not be entitled to all the dividends generated by the

funds underlying assets. Ross (2004) has argued that because of expenses, the fund price should be below its NAV. The larger is the expense ratio paid to management, the larger the discount will be. We find, however, in the fourth row block of Table 2, expense ratios seems to be unrelated to discounts and their persistence.

Finally liquidity can affect a fund's discount. However, it can have both positive and negative impact. On the one hand, illiquid holdings may be difficult to value (Malkiel 1997) and will make arbitrage more costly (Gemmill and Thomas (2002) thus leading to larger discounts. On the other hand, there are clientele effects. The closed-end form may be best suited to illiquid investments and will give small investors an effective means to hold the securities, for example single state municipal bonds. This efficiency gain would lead to smaller discounts (See Cherkes, 2003). From an examination of the funds' holdings, we construct a dummy variable for the presence of illiquid holdings. As shown in the last row block, the evidence seems to be consistent with a weakly positive relationship except in 2002. The evidence also suggests that more illiquid holdings seems to be associated with large persistence in discounts although the evidence is not overwhelming.

Since some of the variables could be correlated with each other, we also examine the explanatory power of all the five variables using multivariate regressions. As shown in Table 3, these rational factor factors can explain about 20% of the cross-sectional differences in discounts (see the adjusted  $R^2$ ). All the variables seems to have the correct signs although only about half are statistically significant. When the five variables are used simultaneously to explain the cross-sectional differences in persistence of discounts, again the most significant variable is the dividend yield.

Insert Table 3 Approximately Here

The evidence presented in this section provides some knowledge regarding both the reasons for the existence of discounts and for their persistence over time. It appears that rational factors can explain at least some share of the cross-sectional variation in discounts and their persistence. Moreover, we have seen that future fund returns can, at least partially, be predicted from both current discounts and NAV return. In the next section, we will further study these relationships as well as examine the

relationship of NAV returns to the performance of different size categories of stocks in general. Finally, we will take a closer look at the differential between the volatility of fund returns and that of their NAV returns.

## 2 Understanding Closed-end Fund Predictability

In this section, we focus on the predictability of fund returns. In order to build a connection between discounts and returns, it is easy to manipulate continuously compounded returns, which are related to the differences in log prices as shown below. Corresponding to this convention, we define the discount ( $\delta_t$ ) of a fund as the natural log of its Net Asset Value (NAV) to its fund price as proposed by Pontiff (1995). In particular, we have,

$$d_t = \ln(V_t) - \ln(P_t) = v_t - p_t, \quad (3)$$

$$r_{p,t} = \ln(P_t + D_t) - \ln(P_{t-1}) \approx \kappa_p + p_t - p_{t-1} + \gamma_p \delta_{p,t}, \quad (4)$$

$$r_{v,t} = \ln(V_t + D_t) - \ln(V_{t-1}) \approx \kappa_v + v_t - v_{t-1} + \gamma_v \delta_{v,t}, \quad (5)$$

where  $\delta_{v,t} = \ln D_t - \ln V_t$ ,  $\delta_{p,t} = \ln D_t - \ln P_t$ ;  $v_t$  and  $p_t$  denote the log fund net asset value (NAV) and the log fund price at time  $t$ , respectively. In the above log-linearization,  $\gamma_v = \frac{D}{D+V}$ ,  $\gamma_p = \frac{D}{D+P}$ ,  $\kappa_v = (1 - \gamma_v) \ln(\frac{D}{V})$ , and  $\kappa_p = (1 - \gamma_p) \ln(\frac{D}{P})$  are steady state parameters. As demonstrated in Campbell (1995), this approximation is quite accurate.

### 2.1 Understanding Predictability by Modeling Discounts Directly

In general, the underlying asset markets and the closed-end fund markets are very different in terms of liquidity and the types of investors who invest in each security (see Cherkes, 2004). The evidence in the previous section suggests that it is reasonable to assume that the log NAV ( $v_t$ ) adjusted for dividend yield follows a random walk with drift process,

$$\begin{aligned} v_t &= \mu_v - \kappa_v + v_{t-1} - \gamma_v \delta_{v,t} + \epsilon_t, \quad \text{or} \\ r_{v,t} &= \mu_v + \epsilon_t, \end{aligned} \quad (6)$$

where  $\mu_v$  is the mean NAV return and  $\epsilon \sim i.i.d(0, \sigma^2)$ . This implies that the NAV return defined in equation (5) has no autocorrelation of any order, that is,  $\rho_{v_t, v_{t+k}} = 0$ . Therefore, we have imposed this minimum structure on the underlying assets.

Since fund returns and discounts are tied through equation (3), we can only model one of them. From the evidence in the previous section, we know that fund returns are somewhat predictable from their past returns. It is easy to see, however, that we will not be able to generate the observed high persistence in fund discounts if we model fund returns using an  $AR(1)$  model. In order to be parsimonious, we directly model the dynamics of discounts by an  $AR(1)$  process instead,<sup>7</sup>

$$d_t - \mu_d = \phi(d_{t-1} - \mu_d) + \eta_t, \quad (7)$$

where  $\mu_d$  is the unconditional mean discount,  $\eta \sim i.i.d(0, \sigma_\eta^2)$ ,  $\sigma_\eta = a\sigma$  and  $\sigma_d^2 = \frac{a^2}{1-\phi^2}\sigma^2$ . Consequently, the  $k$ th autocorrelation in discounts is  $\rho_{d_t, d_{t+k}} = \phi^k$ , which is directly obtained from equation (7). Although this model for discounts is justice from the empirical evidence, most rational explanations discussed in the introduction, such as expense ratios, tax penalties, dividend yields, and liquidity of assets also point to stable discounts. Changes in these rational factors are infrequent and are usually associated with policy changes, which will have long term effects on discounts. In other words, discounts will also be persistent. We will attempt to link discount persistency to rational factors later in the paper. Just as in the ARCH literature that takes volatility persistency as given, we model discount persistence directly in this section.

Although our modeling technique is very simple and different from other studies, it is useful for four reasons. First, by modeling the discount process directly, we not only capture the dynamic properties of discounts, but also are able to explain the predictability of discounts. To show why this is so, we need to determine the return process first. Substituting equation (3) into equation (7), we can write the fund return generating process as,

$$r_{p,t} = \mu_p + [1 - \phi(1 - \gamma_v)](d_{t-1} - \mu_d) + \epsilon_t - (1 - \gamma_v)\eta_t + \bar{d}\delta_{p,t}, \quad (8)$$

where  $\mu_p = \mu_v + \kappa_p - \kappa_v$  and  $\bar{d} = \ln(\frac{V+D}{P+D})$ . Clearly, as long as discounts are stationary, current fund returns can be predicted by the discount of the last period. Since discounts are persistent, all past discounts can predict the current fund return to

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<sup>7</sup>Despite the high persistence of discounts, the augmented Dickey-Fuller test suggests that the series is still stationary.

some degree depending on the persistency. We will show that this feature is exactly what we see in the data. In addition, since dividend yields are persistent, they will contribute to return autocorrelation slightly. Quantitatively, equation (8) suggests that the larger the persistence parameter  $f$ , the smaller the predictability of the first lag. This is a testable implication. Intuitively, when  $f$  is large, a shock to the current discount will be more likely to increase future discounts due to high persistence. High future discounts could be associated with low future returns, which would suggest low predictability of discounts. At the same time, however, the predictability at high lags will drop more slowly. Therefore, understanding why discounts are persistent is fundamental in explaining the predictability of discounts.

Second, shocks to NAVs of the underlying assets have the same impact on fund returns themselves since both the NAVs and the fund prices represent the same claim. The additional shocks that drive fund returns are exactly the same shocks that drive the discounts. It is this additional shock that could drive the “excess volatility” (see Pontiff,1998) of the fund return. As shown below, the additional shocks are only necessary to generate “excess volatility”, but not sufficient. Third, the dominant factor for the predictability of discounts is the persistence of discounts themselves. Since shocks to the discount  $\eta_t$  are *i.i.d.*, and discounts do not predict future NAV return as shown in Table 1, the usefulness of past discounts to predict future return cannot come from predicting  $\epsilon_t$  and  $\eta_t$  in equation (8). In other words, the observed predictability of past discounts is not due to their ability to predicting fundamental shocks to underlying assets. Again, the main source for the predictability of past discounts is from their persistence. This feature is different from the existing explanations. And finally, this linkage for predictability offers a unique opportunity to understand the usefulness of NAV returns to predict the returns from small stock portfolios discussed in the last section.

Despite the weak *i.i.d.* assumption for both  $\epsilon_t$  and  $\eta_t$  individually, we can still allow cross-correlation among the two exogenous shocks in order to generate rich dynamics. Since fund returns can be predicted by their NAV returns (see Day, Li and Xu, 2005), we further assume,

$$Corr(-\eta_t, \epsilon_{t-1}) = c\sigma_\eta\sigma_\epsilon = ac\sigma^2. \quad (9)$$

In other words,  $c$  measures how fund returns respond to past NAV shocks. At the same time, there could be contemporaneous correlation between the discount innovation,  $\eta_t$ , and the NAV return innovation,  $\epsilon_t$ , as shown in the following regression equation,

$$\hat{\eta}_t = 0.0026 + 0.209r_{v,t} - 0.172r_{v,t-1} - 0.049r_{v,t-2} + e_t, \quad R^2 = 28.3\%. \quad (10)$$

(.0270)   (.019)   (0.019)   (0.019)   (.607)

Clearly, contemporaneous correlation is important and high order cross-correlations (more than one lag) are negligible. Therefore, we further assume that,

$$\text{Corr}(-\eta_t, \epsilon_t) = q\sigma_\eta\sigma_\epsilon = aq\sigma^2. \quad (11)$$

As supported by equation (10) and empirical evidence, we continue to assume that  $\text{Corr}(-\eta_t, \epsilon_{t-k}) = 0$ , for  $k \geq 2$ . Equations (9) and (11) allow us to capture first order autocorrelation in fund returns and first order cross-correlation between NAV returns and future fund returns.

Applying equations (6) to (11), we can derive the following correlations among  $r_{p,t}$ ,  $r_{v,t}$ , and  $d_t$ .

$$\rho_{d_t, r_{p,t+k}} = a\phi^{k-1} \sqrt{\frac{1 - \phi_1}{(1 + \phi)(1 - 2a\theta)}} \quad (12)$$

$$\rho_{r_{v,t}, r_{p,t+k}} = \begin{cases} a \frac{c+q(1-\phi_1)}{\sqrt{1-2a\theta}} & k = 1 \\ a \frac{(1-\phi_1)\phi^{k-1}(q\phi-c)}{\sqrt{1-2a\theta}} & k \geq 2 \end{cases} \quad (13)$$

$$\rho_{r_{p,t}, r_{p,t+k}} = \begin{cases} a \frac{c+(1-\phi_1)\theta}{1-2a\theta} & k = 1 \\ a(1-\phi_1)\phi^{k-2} \frac{c+\phi\theta}{1-2a\theta} & k \geq 2 \end{cases} \quad (14)$$

where  $\theta = q - \frac{a}{1+\phi}$  and  $\phi_1 = \phi(1 - \gamma_v)$ . Equation (12) suggests that the predictability is indeed due to the persistence of discounts. Although  $(1 - \phi_1)$  might be small,  $\sqrt{1 - \phi_1}$  could be substantial. For example, when  $\phi = 0.95$  and  $\gamma_v = 0.05$ ,  $\sqrt{1 - \phi_1}$  is over 31%. Moreover, the predictability decays slowly since  $\phi$  is very close to one. For the same  $\phi$ , the predictability drops by 46% after a quarter. We also see that the first order cross-correlation between NAV return and the next period fund return is directly related to correlation between current innovation in the discount and past NAV shocks. At the same time, there is substantial first order auto-correlation in the fund return itself, but the magnitude is relatively small compared to the cross-correlation.

## 2.2 Simulating the Predictability of Discounts

Given the explicit formula for auto and cross correlations, we can compute various correlations at different lags. This calibrated correlation structure can then be compared to the actual correlation structure. From the actual aggregate returns and discounts, we plot the autocorrelation for aggregate fund returns and aggregate NAV returns, the cross-correlation between discount and future fund return, as well as the cross-correlation between NAV returns and future fund returns. These are shown in Figure 1 over the ten year period from 1993 to 2002. First, the autocorrelations for NAV returns seems to be small and fluctuate between  $-5\%$  to  $5\%$ . Therefore, the markets for the underlying assets do seem to be efficient as mentioned above. In contrast, the auto-correlation for fund returns exceeds  $20\%$  at the first lag. Starting from the third lag, fund return autocorrelations are very similar to those of NAV returns. This suggests that the autocorrelation in returns will be much weaker in the low frequency return data, such as monthly or quarterly data. In addition, the cross-correlation between lagged NAV returns and current fund returns is close to  $25\%$ . Again, when the number of lags is greater than three, the cross-correlations are very similar to those of the autocorrelations of returns.

Insert Figure 1 Approximately Here

Figure 1 also reveals that not only past discounts predict current fund return, but the predictability at different lags seems to be very similar. Perhaps this should be expected since discounts are very persistent. If there is predictability at a certain lag, we should see predictability at most of the lags. This is yet another reason to model the discount process directly.

Using the summery statistics in Table 1, we set  $\phi = 0.98$ ,  $\sigma = \sigma_\epsilon = 1.42\%$ , and  $\sigma_\eta = .72\%$ . Therefore, we have  $a = \sigma_\eta/\sigma_\epsilon = 0.50$ . The estimate for  $c$  can be obtained from equation (2), i.e.,  $c = 0.17 * \sigma_v/\sigma_\eta = 0.17 * 1.42/.72 = 0.34$ . We have also computed the contemporaneous correlation between discount innovations and NAV innovations to be  $q = 0.39$ . In addition, we estimate  $\gamma_v$  to be  $0.3\%$ . Plugging these parameters into equations (12) to (14), we obtain various autocorrelations and cross-

correlations among fund returns, NAV returns, and discounts. The results are shown in Figure 2.

Insert Figure 2 Approximately Here

First, the general pattern in the correlation structure appears to be very similar to those shown in Figure 1. In particular, the predictive power of the current discount for future returns decreases very slowly as future periods increase. However, we are only able to explain about a third of the predictability level observed in the data. If we want to match the observed 15% correlation between past discounts and current fund returns, the persistence coefficient needs to come down to 0.94. At the same time, the predictive power of both past NAV returns and past fund returns for current fund return is substantial at the first lag. The magnitude of these calibrated correlations matches over 60% of those shown in Figure 1. These correlations also die down to zero at the second lag. Of course, what has been shown in Figure 1 only represents one realization, while results displayed in Figure 2 represent the true structure if the parameters used are the true ones.

Our model is based on simple assumptions. One of them is an AR(1) process for discounts. Actually, aggregate discounts are best fit by an AR(2) model as in the following regression equation,

$$d_t = 0.144 + 0.760d_{t-1} + 0.223d_{t-2} + \hat{\eta}_t, \quad R^2 = 96.3\%. \quad (15)$$

(.078)   (.043)        (0.043)   (0.698)

Therefore, we can recalculate the correlation structure using a simulation approach. In particular, we first generate log NAV according to equation (6) with an initial value set to 110. For simplicity, we assume that the NAV process is dividend reinvested in this part of the exercise. We then generate discount innovations according to (10). Discounts are easily computed from equation (15) with an initial value of 10 to reflect the average discount level. These numbers can then be used to generate fund prices according to equation (8) with an initial value of 100. Using these simulated data, we compute the corresponding auto-correlations and cross-correlations. This process is repeated 1000 times. The average correlation structure is reported in Figure 3.

Insert Figure 3 Approximately Here

Surprisingly, not only the cross-correlation between lagged NAV returns and current fund returns, but also the autocorrelation in fund returns are almost the same as those shown in Figure 1. Different from what would be expected according to the AR(1) structure for discounts, we continue to see substantial cross-correlation and auto-correlation at the second lag, which closely resembles those shown in Figure 1. The predictability of discounts for future returns is similar to that shown in Figure 2. Therefore, our model is fairly robust with respect to the discount process in generating the observed correlation structure.

In all these exercise, persistence of discounts plays an important role. As discussed in the last section, our model implies a negative relationship between persistency level and the ability of discounts to predict future returns. Therefore we run a simple cross-sectional regression of the following,

$$\rho_{(d,r_{+1}),i} = 0.962 - 0.836\rho_{d,i} + u_t, \quad R^2 = 48.4\%. \quad (16)$$

(.106)   (.114)   (0.049)

where  $\rho_{(d,r_{+1}),i}$  is  $i$ -th fund's cross-correlation between current discount and future fund return. Clearly the negative relationship is very significant. Discount persistence explains almost 50% of differences in the discount predictability for future returns of individual funds. This evidence indirectly supports our efforts to model the discount process directly. Therefore, we further investigate the issue of what determines the persistence of discounts in the next section.

## 2.3 Predicting Closed-end Fund Returns from Portfolio Returns

In the previous section we have made some progress in pinning down the role of the persistence of discounts in explaining future fund returns. Simulation results suggests that a large portion of the cross-correlation between current discounts and future fund returns can be explained by the persistence of discounts itself. Since we linked discount persistence to several rational and relatively constant explanations, such as expense ratios, dividend yields, unrealized capital gains, liquidity, and turnover through a cross-sectional study in section 1.2, the predictability of discounts might be rational.

Closed-end fund returns not only can be predicted by discounts, but more significantly by NAV returns as shown in section 1.1. At the same time, however, current fund returns do not predict future NAV returns. Day, Li, and Xu (2005) relate such predictability to adaptive expectation of investors on individual fund level. In this section, we offer an alternative explanation similar to Lo and MacKinlay's (1990) findings that large stock portfolio returns predict small stock portfolio returns. In fact, most equity closed-end funds are invested in large stocks or stocks with similar characteristics to large company stocks. The average market capitalization of assets that a typical fund equity holds is about 12 billion dollars. Therefore, NAV returns exhibit no more persistence than large stock returns. Most closed-end fund investors tend to be small investors. Their investment behavior may likely be subject to "investors sentiment" risk of Lee, Shleifer, and Thaler (1991). Such behavior may also be shared by investors in small stocks. In other words, fund returns may well behave more like small stock returns rather than large stock returns. If this analogy is reasonable, NAV returns (made up returns of large-company stocks) should also predict future closed-end fund returns.

In order to explore this linkage, we obtain six size and book-to-market (B/M) sorted portfolio returns.<sup>8</sup> These daily time series are compounded to weekly returns with the same ending week dates as our closed-end fund data. We first investigate

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<sup>8</sup>We are grateful to Kenneth French for making this data set available on his website.

whether fund returns are associated with small stock portfolio returns while NAV returns are related to the large portfolio returns. Although this should certainly be the case given the average size of individual assets that each fund holds, we need to be sure that this holds in our data set. For this reason, we take an indirect approach by regressing fund returns and NAV returns on size portfolio returns, respectively. The results are reported in Panel A of Table 3.

Insert Table 4 Approximately Here

Since each of series under investigation is highly correlated with the market factor, we first orthogonalize all the series to market returns when reporting the contemporaneous relationship in Panel A. Although we have six portfolios, we ignore the middle book-to-market portfolio in each size group. In other words, we study portfolio  $r_{sl}$ ,  $r_{sh}$ ,  $r_{bl}$ , and  $r_{bh}$ , where ‘s,’ ‘b,’ ‘l,’ and ‘h,’ denote small size, big size, low B/M, and high B/M, respectively.<sup>9</sup> After extracting the market components, we find that fund returns are highly correlated with the small size portfolio with high book-to-market value. The explanatory power exceeds 23% in terms of adjusted  $R^2$ . When fund returns are regressed on the returns from portfolios of large-capitalization stocks, the total explanatory power is just half of that using the returns from portfolios of small-capitalization stocks although both  $r_{bl}$  and  $r_{bh}$  are significant. This evidence suggests that fund returns more resemble the small portfolio returns than large portfolio returns. When all the four portfolios are included in the regression, it also suggests that fund returns are also closely related to portfolio with high book-to-market value. In contrast, the NAV returns seem more related to large stock portfolios than to small stock portfolios not only in terms of  $R^2$ s (14.3% versus 11.9%), but also in terms of the significance of each variable in the multiple regression shown in the sixth equation of Table 3.

The above evidence indicates that we need to investigate if the predictability of NAV returns for future fund returns is related to the predictability of large stock

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<sup>9</sup>Closed-end funds usually hold much more illiquid securities than open-end funds. These illiquid assets tend to have high book-to-market ratio. Therefore, we retain the book-to-market portfolios in our study.

returns for future small stock returns. As a comparison, we first regress the aggregate fund returns on past discounts. It is significant with an adjusted  $R^2$  of 2.3% as shown in the first equation of Panel B of Table 4. When fund returns are regressed on both past fund returns and past NAV returns, only past NAV returns is significant, but the adjusted  $R^2$  jumps to 5.6%. One might suspect that such predictability is due to a market micro-structure effect. Boudoukh, Richardson, and Whitelaw (1994) have shown that most of the cross-serial correlation between small portfolio returns and lagged large portfolio returns can be explained by autocorrelation in small portfolio returns, while autocorrelation in small portfolio returns can in turn be explained by nonsynchronous trading. If this is also the case here, we should see that only the lagged fund return variable should be significant. However, just the opposite is the case. This evidence also suggests that it is unlikely that the predictability is due to “investor’s sentiment” risk since the past fund return variable is insignificant. However, it is consistent with the story that investors adjust their expectations according to NAV information. When all three variables are included in the regression, the discount variable is still significant with an adjusted  $R^2$  of 6.9%. Therefore, discounts might have summarized some of the dynamic features of both fund returns and NAV returns that are not a simple linear combinations of the two.

When portfolio returns are used as a predictor,  $r_{sl}$  predicts future closed-end fund returns with an adjusted  $R^2$  of 4.6%. What is more surprising is that large portfolio returns can predict closed-end fund returns with an even bigger adjusted  $R^2$  of 8.6%. Market micro-structure effects cannot play a significant role here since the predictability comes largely from the returns of portfolios with large capitalization stocks confirmed by a multivariate regression using all four portfolios returns (not shown in the Table). Therefore, a large part of the predictable component in the fund returns is indeed due to the general phenomenon of large stocks leading small stocks. This is even more apparent when using past discounts, past fund returns, past NAV returns, and past large portfolio returns at the same time. As the last equation in Panel B of Table 3 shows, the past discount and large portfolio returns are still significant, while fund returns and NAV returns are insignificant at a 5% level. This suggests that the predictability of past NAV returns for future fund returns is due to the correlation between NAV returns and large stock portfolio returns.

These results provide further evidence on closed-end fund return predictability. It seems that there are at least two predictable components in the closed-end fund returns. The first component is due to persistence in discounts; and the second component is related to the predictability of large stocks for small stocks as has been observed in the general equity market. None of these two components seem to predict future NAV returns, a result similar to the finding that returns from small or large stock portfolios do not predict future large stock portfolio returns. Market microstructure effects do not appear to contribute to this predictability since past fund returns themselves are insignificant once other variables are added.

The first source of predictability also suggests that discounts might reflect some unique information about future equity returns. In fact, Swaminathan (1996) has shown that closed-end fund discounts forecast future excess returns of small firms. We reexamine the issue using our more recent sample for small stock portfolio with high book-to-market ratio ( $r_{sh}$ ), since this portfolio ties more closely to closed-end fund returns. Indeed, discounts predict  $r_{sh}$  in our sample too (not shown in the table) but with an  $R^2$  less than 1%. However, when past returns for the small stock portfolio with high B/M ( $r_{sh}$ ) are also used to control for possible micro-structure effects, the predictive power of discounts vanishes as shown in the first equation of Panel C of Table 3.<sup>10</sup> In other words, the predictability of discounts is very much unique to closed-end funds. This result seems to be consistent with Swaminathan's (1996) effort to link the information in discounts to expectations of future earnings growth and expectations of future inflation.

As a final exercise we examine if NAV returns predicts small stock portfolio returns. This is carried out in the second equation of Panel C of Table 3. Indeed, NAV returns predicts future small stock portfolio returns, even after controlling for the predictive power from returns of small stock portfolios. The two variables explain close to 11% of the total variations in the future small stock returns. Such predictive power might be attributed to liquidity factors that influence both closed-end funds' underlying assets and small stock returns. Generally speaking, small stocks are less

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<sup>10</sup>As Swaminathan (1996) suggests that information in discounts is independent of those commonly used forecasting variables such as the dividend yield on the market, the default spread, and the term spread, we do not include these other variables.

liquid than large stocks. At the same time, many closed-end funds carry illiquid positions. If it is reasonable to assume that liquidity is persistent, shocks to liquidity will have long term effects. Thus, NAV returns can predict small stock returns if both are subject to the same liquidity shock, which suggests that the predictability evidence on closed-end funds and small stocks is consistent with the efficient-market hypothesis.

### 3 Another Look at “Excess Volatility”

An equally important issue that is related to market efficiency is the volatility issue. Shiller (1981) argued that the common stock prices were too volatile relative to the dividends and cash flows of the underlying corporations. This so-called “excess volatility” was used by Shiller as an evidence against the “Efficient Market Hypothesis.” Since observed stock prices reflect investors’ expectation about future cash flows, they should have lower or equal volatility than prices constructed from realized cash flows if investors are rational.<sup>11</sup> Shiller’s evidence points to the opposite.

In the case of closed-end funds, one can naturally consider NAV as the realized value for a closed-end fund (see Pontiff, 1996). Therefore, a fund’s NAV should be more volatile than the fund price. Pontiff has found the opposite to be true, and has suggested that the pricing of closed-end fund shares exhibits “excess volatility,” especially for individual funds. In this section, we first investigate under what condition “excess volatility” might occur in our model framework of section 2.1. We then study the issue from both individual fund level as well as aggregate level. We also provide evidence on time-varying conditional volatilities for both fund returns and NAV returns using a GARCH framework.

#### 3.1 A Condition for “Excess Volatility”

In general, “excess volatility” in closed-end funds does not necessarily signal market inefficiency if there exist additional factors that determine fund prices. In our framework, this is easily understood by examining equations (6) and (8). We study return volatility in an equivalent manner as in Pontiff (1996). Compared with NAV returns, fund returns shown in equation (8) should be subject to additional shocks, namely the shock to discounts. In other words, shocks to discounts summarize all the possible shocks to closed-end fund returns other than the shocks to the NAVs

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<sup>11</sup>If we consider the current stock price is the best forecast for future discounted cash flows, the price constructed from realized cash flows should be equal to the forecast plus the forecast error. In other words, the volatility of the realized prices should be higher than that of the ex ante price.

themselves. It should be possible to observe “excess volatility” when they are due to additional shocks. Of course, if these additional shocks are primarily due to rational factors discussed in section 1.2, the closed-end fund market would still be efficient even when there is “excess volatility.”

The above analysis ignores the possible correlation between discount shocks and NAV shocks. From the definition of discounts in equation (3), it is clear that an NAV shock will cause discount to rise. As shown in equation (10), the relationship is indeed positive and significant. This correlation will reduce the overall return volatility of the fund. In particular, we compute the ratio between fund return volatility and the NAV return volatility as,

$$\frac{\sigma_{r_{p,t}}}{\sigma_{r_{v,t}}} = 1 - 2a\left(q - \frac{a}{1+\phi}\right). \quad (17)$$

Clearly, when  $q = 0$ , i.e., there is no contemporaneous cross-correlation between shocks to NAV returns and shocks to discounts, fund return volatility will exceed that of the NAV return, i.e., we will discover an “excess volatility” phenomenon.

When there are multiple shocks, the correlations among them could also play an important role. For example, if NAV shocks and discount shocks are highly correlated, the volatility contribution from additional shocks could be largely offset by the correlation. In fact, when  $q > \frac{a}{1+\phi}$ , “excess volatility” disappears. This is exactly what we have seen in Table 1. In other words,  $\frac{a}{1+\phi}$  is the lower bound on this contemporaneous correlation that will be large enough to offset the volatility of additional shocks. Therefore, closed-end funds will not necessarily exhibit “excess volatility.”

### 3.2 Aggregate versus Individual Level Volatilities

The “excess volatility” result for closed-end funds shown by Pontiff (1996) is largely based on comparing volatilities of fund returns versus volatilities of NAV returns for individual closed-end funds. We have replicated his results in Panel A of Table 5. The average annualized volatility across individual funds estimated from weekly fund returns is 19.6%. This volatility estimate is much higher than that of NAV returns, which is 14.4%. If we treat the variance ratio between fund return and NAV return,

$\frac{\hat{\sigma}_p^2}{\hat{\sigma}_v^2}$ , as approximately an  $F$  distribution, the hypothesis of no “excess volatility” is rejected with a  $p$ -value close to zero.

Insert Table 5 Approximately Here

We have also summarized the distribution of fund return volatilities and NAV return volatilities in the same panel. Clearly the distribution of fund return volatilities is shifted to the left of the distribution of NAV return volatilities. Moreover, by comparing median with mean volatility, we conclude that all volatility distributions are skewed to the right since the means are greater than the corresponding medians. In other words we are not only more likely to see large volatilities but also return volatilities are more likely to be greater than the corresponding NAV return volatilities.

Just like individual stocks, individual closed-end funds’ volatilities also include idiosyncratic volatilities. Therefore, the volatilities of the closed-end fund market as a whole may behave differently. As shown in Table 1 and reproduced in Panel B of Table 5, the annualized volatility of aggregate NAV returns (10.22%) is larger than that of the aggregate fund returns (9.47%). The approximate  $F$  test now suggests that we can reject the hypothesis of “excess volatility” for the closed-end fund market as a whole.

Since we have estimated volatility from weekly returns, market micro-structure effects might be a concern. In order to avoid this issue, we recompute annualized volatilities based on bi-weekly returns, monthly returns (approximated by 4 weeks returns), quarterly returns, and semi-annual returns. The corresponding volatilities are also reported in Panel B of Table 5. In all the cases, aggregate fund return volatilities are all below aggregate NAV return volatilities, except for the semi-annual case. Even for the semi-annual case, the difference is insignificant, however. Therefore, the conclusion of no “excess volatility” on aggregate level is robust.

The overall evidence suggests that there might be some inefficiency in the trading of individual closed-end funds, which causes the phenomenon of “excess volatility.” The impact of the inefficiency tends to be idiosyncratic, however. For example, if

there exists ‘investors’ sentiment’ risk in the closed-end fund market, this sentiment is more likely to make investors overly optimistic about one group of funds while overly pessimistic about other funds. Thus, the overall market is still efficient.

### 3.3 The Time-varying Volatility

As we found no evidence of “excess volatility” on the aggregate level, one possible reason discussed in section 3.1 is that additional pricing factors for closed-end funds might be negatively correlated with the market factor, which offsets the systematic volatility component in the total volatility. This suggests that the unconditional estimates of volatility may not tell the whole story. In this section, we estimate the conditional volatility using the most popular GARCH(1,1) model for both aggregate fund returns and aggregate NAV returns. Based on demeaned return series, we have the following estimates,

$$h_{p,t+1} = 0.215 + 0.763h_{p,t} + 0.110u_{p,t}^2, \quad (18)$$

(.021)   (.018)   (0.010)

$$h_{v,t+1} = 0.171 + 0.787h_{v,t} + 0.135u_{v,t}^2, \quad (19)$$

(.014)   (.012)   (0.011)

where  $h_{p,t}$  and  $h_{v,t}$  are conditional volatilities for fund returns and NAV returns, respectively, while  $u_{p,t}$  and  $u_{v,t}$  are the demeaned fund returns and NAV returns, respectively.

From equations (18) and (19), we can compute the unconditional volatility for fund returns to be  $\sigma_p = \sqrt{\frac{0.215}{1-(0.763+0.110)}} = 1.301$ , while the unconditional volatility for NAV returns to be  $\sigma_p = \sqrt{\frac{0.171}{1-(0.787+0.135)}} = 1.480$ . In other words, we again find that aggregate fund returns are less volatile than the volatility of the underlying assets. The overall closed-end fund market then appears to be efficient. Such a finding does not challenge Pontiff’s results since he only suggests that irrationality may be present in the pricing of the shares of individual closed-end investment companies. We want to emphasize that any such inefficiency seems to be idiosyncratic, however, given our evidence on aggregate level. In addition, since the persistence parameter from

equation (18) is  $0.873 = (0.763 + 0.110)$ , which is much lower than that from (19) of  $.922 = (0.787 + 0.135)$ , the conditional volatility of fund return is less persistent than that of the NAV return. In other words, the impact of a volatility shock will die out more quickly for fund returns than for NAV returns. This evidence suggests that any unique factors to closed-end funds are less likely to have permanent effects to fund prices.

One of the important findings of Campbell, Lettau, Malkiel, and Xu (2001) is that while the market volatility seems to be stable over time, there has been a significant upward trend in the idiosyncratic volatility. It is also interesting to see if there is a trend in the conditional volatility. We estimate the two volatility series from equations (18) and (19) and plot them in Figure 4.

Insert Figure 4 Approximately Here

First, there are occasional periods where we do observe “excess volatility.” The conditional volatilities of fund returns dominate the conditional volatilities of NAV returns most of time. Second, there are apparent upward trends in both conditional volatilities. Yet, the upward trend could simply be due to high persistence in the series themselves. Therefore, we run the following regressions to test the significance of the trend.

$$\hat{\sigma}_{p,t+1} = 0.131 + 0.0001t + 0.869\hat{\sigma}_{p,t} + 0.006\hat{\sigma}_{v,t} + e_{p,t+1}, \quad (20)$$

(.025) (.00004) (0.038) (0.033)

$$\hat{\sigma}_{v,t+1} = 0.137 + 0.0002t + 0.847\hat{\sigma}_{v,t} + 0.018\hat{\sigma}_{p,t} + e_{v,t+1}, \quad (21)$$

(.030) (.00005) (0.039) (0.045)

where  $\hat{\sigma}_{p,t} = \sqrt{h_{p,t}}$  and  $\hat{\sigma}_{v,t} = \sqrt{h_{v,t}}$ . Since there is no unit root in either of the volatility series, the time trends are very significant in both equations. In other words, both fund returns and the NAV returns have become increasingly volatile in recent years. This is consistent with the Campbell, et al. (2001) findings. Moreover, the time trend in NAV return volatility is twice as large as that in the fund return volatility. This suggests that investing in closed-end investment company shares may actually be even safer than investing in open-end funds.

## 4 Concluding Comments

The closed-end fund discount phenomenon has fascinated investigators for many years. Several attempts have been made from both rational and irrational perspectives to explain the causes of discounts. We now understand that several factors do affect discounts including dividend yields, unrealized capital gains, turnover, expense ratios, illiquid assets, etc. It is equally challenging, yet less attention has been paid, to understand the predictability of discounts for fund future returns. In this paper, we take an indirect approach by modeling the discount process as a persistent AR(1) process. It is shown that much of the predictability of discounts for future fund returns can be generated as a result. Moreover, fund returns themselves will be auto correlated while the underlying asset returns remain independent over time. This suggests that it is also important to understand the persistence of discounts.

If the documented rational factors are the main driving force for the discounts, discounts will not only be relatively stable over time but also be persistent. This is because factors such as expense ratios and illiquid holdings do not change often, and managers try to maintain a stable dividend yield. Thus any shocks to these factors will have long-term effects. Our evidence supports this hypothesis.

Although the closed-end fund market is relatively small compared to open-end fund market, the behavior of the market has important implications for market efficiency. Investigating the difference between fund price volatility and its NAV volatility has been used to suggest a market inefficiency. Pontiff (1996) has found evidence that fund returns are more volatile than their NAV returns, which suggests a market inefficiency in the spirit of the “excess volatility” work of Shiller. We found the opposite on the aggregate level, which suggests that investors may be overly enthusiastic about some funds and lose interest in others, but the market as a whole could still be efficient.

In addition, we find that NAV returns of closed-end funds predicts fund returns, but not the other way around. This evidence resembles the lead-lag relationship between large-cap portfolio returns and small-cap portfolio returns documented by Lo and MacKinlay (1990). In fact, we show that large-cap portfolio returns capture a

large portion of the predictable components of the future fund returns. Such predictability is unlikely to be explained by market microstructure effects. Finally, the NAV returns seem to have predictable power for small-cap portfolio returns. This is a useful extension of Swaminathan's (1996) finding that discounts predict small-capitalization portfolio returns.

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Table 1: **Characteristics of Aggregate Closed-end Equity Funds**

This table summarizes the statistics for equal weighted U.S. equity closed-end funds. The results are based on weekly data from January 1993 to December 2002.  $d_t$ ,  $r_{v,t}$ ,  $r_{p,t}$ , and  $r_{m,t}$  are the log discount, the log NAV return, the log fund return, and the value weighted market index return at time  $t$ , respectively.  $\rho$  denotes the first order auto-correlation for each series. For univariate regressions, each column represents different regression. All of the numbers in the table are in percentage form.

	$d_t$	$r_{v,t}$	$r_{p,t}$	$r_{m,t+1}$
Mean	8.42	0.139	0.145	0.187
Std Error	3.62	1.42	1.31	2.32
$\rho$	98.04	4.53	22.59	-7.31
Univariate Regression with Dependent Variable $r_{v,t+1}$				
Coefficient	0.036	0.045	0.078	0.558
( $Std_{Coef}$ )	(0.017)	(0.044)	(0.047)	(0.011)
$\sigma_{residual}$	1.413	1.417	1.415	0.580
$R^2$	0.829	0.206	0.521	83.31
Univariate Regression with Dependent Variable $r_{p,t+1}$				
Coefficient	0.057	0.225	0.226	0.427
( $Std_{Coef}$ )	(0.016)	(0.040)	(0.043)	(0.016)
$\sigma_{residual}$	1.298	1.275	1.281	0.863
$R^2$	2.45	5.92	5.11	56.92
Univariate Regression with Dependent Variable $d_{t+1}$				
Coefficient	0.980	0.245	0.23	0.197
( $Std_{Coef}$ )	(0.009)	(0.112)	(0.121)	(0.068)
$\sigma_{residual}$	0.715	3.611	3.616	3.599
$R^2$	96.1	0.921	0.695	1.58

Table 2: **What Determines the Level and Persistence of Closed-end Fund Discounts?**

This Table reports the univariate cross-sectional regressions of regressing average discount ( $\bar{d}$ ), or persistence of discount ( $\rho_d$ ), on annual dividend yield ( $D/P$ ), future tax obligation ( $TAX$ ), expense ratio ( $EXP$ ), or annual turnover ( $TN$ ), or an illiquidity dummy ( $ILQD$ ), respectively for each year from 1999 to 2002. ‘\*\*’ and ‘\*’ represent 5% and 10% significance levels, respectively.

	Dependent Variable: $\bar{d}$				Dependent Variable: $\rho_d$			
	1999	2000	2001	2002	1999	2000	2001	2002
$D/P$	-0.418*	-0.155	-0.194	-0.420*	1.204**	1.142*	0.974**	1.088**
Std Error	0.231	0.223	0.274	0.255	0.475	0.625	0.455	0.296
$R^2$	6.533	1.037	1.123	6.075	12.01	6.759	9.450	24.38
$TAX$	0.100**	0.016	0.271**	0.226**	-0.408**	-0.257*	-0.478**	-0.233**
Std Error	0.051	0.054	0.076	0.066	0.105	0.156	0.130	0.090
$R^2$	7.733	0.189	22.59	21.72	25.12	5.811	23.43	13.79
$TN$	-2.711**	-0.502	-4.974**	-1.648	7.645**	5.029	7.405**	3.219**
Std Error	1.296	1.274	1.849	1.312	2.653	3.615	3.269	1.656
$R^2$	8.524	0.336	14.12	3.619	15.02	4.039	10.44	8.259
$EXP$	0.511	2.200	0.692	-0.425	7.640	-7.893	0.012	0.030
Std Error	2.637	2.228	3.095	2.764	5.493	6.406	5.363	3.576
$R^2$	0.080	2.076	0.114	0.056	3.953	3.194	0.000	0.000
$ILQD$	3.920	5.076**	0.472	-2.689	13.80**	11.16	10.85*	-1.785
Std Error	2.723	2.394	3.818	3.526	5.581	7.035	6.705	4.256
$R^2$	4.059	8.729	0.034	1.305	11.09	5.079	5.495	0.398

Table 3: **What Determines the Level and Persistence of Closed-end Fund Discounts?**

This table reports the multivariate cross-sectional regressions of regressing average discount ( $\bar{d}$ ) or persistence of discount ( $\rho_d$ ) on annual dividend yield ( $D/P$ ), future tax obligation ( $TAX$ ), expense ratio ( $EXP$ ), annual turnover ( $TN$ ), and an illiquidity dummy ( $ILQD$ ) for each year from 1999 to 2002. ‘\*\*’ and ‘\*’ represent 5% and 10% significance level, respectively.

Year	<i>Constant</i>	<i>D/P</i>	<i>TAX</i>	<i>EXP</i>	<i>ILQD</i>	$R^2$
Panel A: Dependent Variable: $\bar{d}$						
1999	5.685	-0.189	0.157**	1.456	6.901**	15.20
Std Error	5.474	0.226	0.056	2.648	2.739	
2000	8.756	-0.121	0.118**	1.072	9.096**	17.30
Std Error	4.693	0.212	0.056	2.217	2.626	
2001	-5.733	-0.136	0.382**	2.543	8.709**	27.50
Std Error	4.813	0.234	0.084	2.716	3.806	
2002	4.327	-0.230	0.224**	0.417	1.110	16.20
Std Error	4.387	0.256	0.082	2.555	3.670	
Panel B: Dependent Variable: $\rho_d$						
Persistence 1999	59.17**	1.335**	-0.328	3.147	7.519	34.20
Std Error	10.95	0.453	0.113	5.296	5.478	
2000	73.98**	0.905	-0.269	-8.353	5.578	8.90
Std Error	14.55	0.658	0.175	6.875	8.142	
2001	75.29**	0.905**	-0.493**	-5.233	-0.360	26.30
Std Error	8.405	0.409	0.148	4.742	6.647	
2002	71.48**	0.863**	-0.230**	-0.935	-4.751	26.90
Std Error	5.299	0.309	0.099	3.086	4.433	

Table 4: **Predicting Closed-end Fund Return from Portfolio Returns**

This table reports tests of the predictability of equal weighted U.S. equity closed-end funds. The results are based on weekly data from January 1993 to December 2002.  $d_t$ ,  $r_{v,t}$ ,  $r_{p,t}$ , and  $r_{m,t}$  are the log discount, the log NAV return, the fund return, and the value weighted market index return at time  $t$ , respectively.  $r_{sl,t}$ ,  $r_{sh,t}$ ,  $r_{bl,t}$ , and  $r_{bh,t}$  are size and book-to-market (B/M) sorted portfolio returns obtained from Kenneth French's website, where 's,' 'b,' 'l,' and 'h,' denote small size, big size, low B/M, and high B/M, respectively. In panel A, all the variables are orthogonal to the market returns.  $R^2$  is the adjusted coefficient of determination.

Dep. Var.	$d_t$	$r_{p,t}$	$r_{v,t}$	$r_{sl,t}$	$r_{sh,t}$	$r_{bl,t}$	$r_{bh,t}$	$R^2$
Panel A. Contemporaneous Relationship								
$r_{p,t}$ (Std.E)				-0.017 (0.028)	0.350 (0.045)			23.4
$r_{p,t}$ (Std.E)						0.090 (0.031)	0.214 (0.028)	11.6
$r_{p,t}$ (Std.E)				0.056 (0.035)	0.219 (0.052)	-0.016 (0.035)	0.174 (0.028)	28.5
$r_{v,t}$ (Std.E)				0.037 (0.020)	0.104 (0.032)			11.9
$r_{v,t}$ (Std.E)						0.118 (0.020)	0.130 (0.018)	14.3
$r_{v,t}$ (Std.E)				0.057 (0.025)	0.038 (0.037)	0.058 (0.024)	0.127 (0.020)	20.2
Panel B. Predicting Fund Returns ( $r_{p,t+1}$ )								
$r_{p,t+1}$ (Std.E)	0.056 (0.015)							2.3
$r_{p,t+1}$ (Std.E)		0.058 (0.086)	0.178 (0.080)					5.6
$r_{p,t+1}$ (Std.E)	0.043 (0.015)	0.062 (0.086)	0.156 (0.080)					6.9
$r_{p,t+1}$ (Std.E)				0.093 (0.044)	0.004 (0.077)			4.6
$r_{p,t+1}$ (Std.E)						0.059 (0.027)	0.126 (0.040)	8.6
$r_{p,t+1}$ (Std.E)	0.046 (0.015)	0.062 (0.086)	-0.191 (0.118)			0.103 (0.040)	0.147 (0.044)	10.0
Panel C. Predicting Portfolio Returns								
$r_{sh,t+1}$ (Std.E)	0.020 (0.020)						0.312 (0.041)	9.8
$r_{sh,t+1}$ (Std.E)			0.198 (0.074)		0.198 (0.060)			10.8

Table 5: **Aggregate versus Individual Level Volatilities**

This table provides both the distribution for annualized weekly volatility estimates of individual closed-end funds and aggregate return volatility estimates at different frequencies from January 1993 to December 2002.  $\sigma_v$  and  $\sigma_p$  are volatilities of the log NAV return and the log fund return, respectively.

Panel A: Volatilities of Individual Funds					
	Mean	Std.D	10%	50%	90%
$\sigma_v$	14.40	5.91	7.71	13.40	21.60
$\sigma_p$	19.60	6.80	13.40	17.30	28.20
$\sigma_p^2/\sigma_v^2$	2.16	0.23	0.99	1.69	4.08
Panel B: Aggregate Volatilities at Difference Frequencies					
	1 Week	2 Weeks	4 Weeks	13 Weeks	26 Weeks
$\sigma_v$	10.22	7.64	5.80	3.63	1.87
$\sigma_p$	9.47	7.58	5.79	3.52	2.14
$\sigma_p^2/\sigma_v^2$	0.86	0.98	1.00	0.94	1.30
# of Observations	520	260	130	40	20

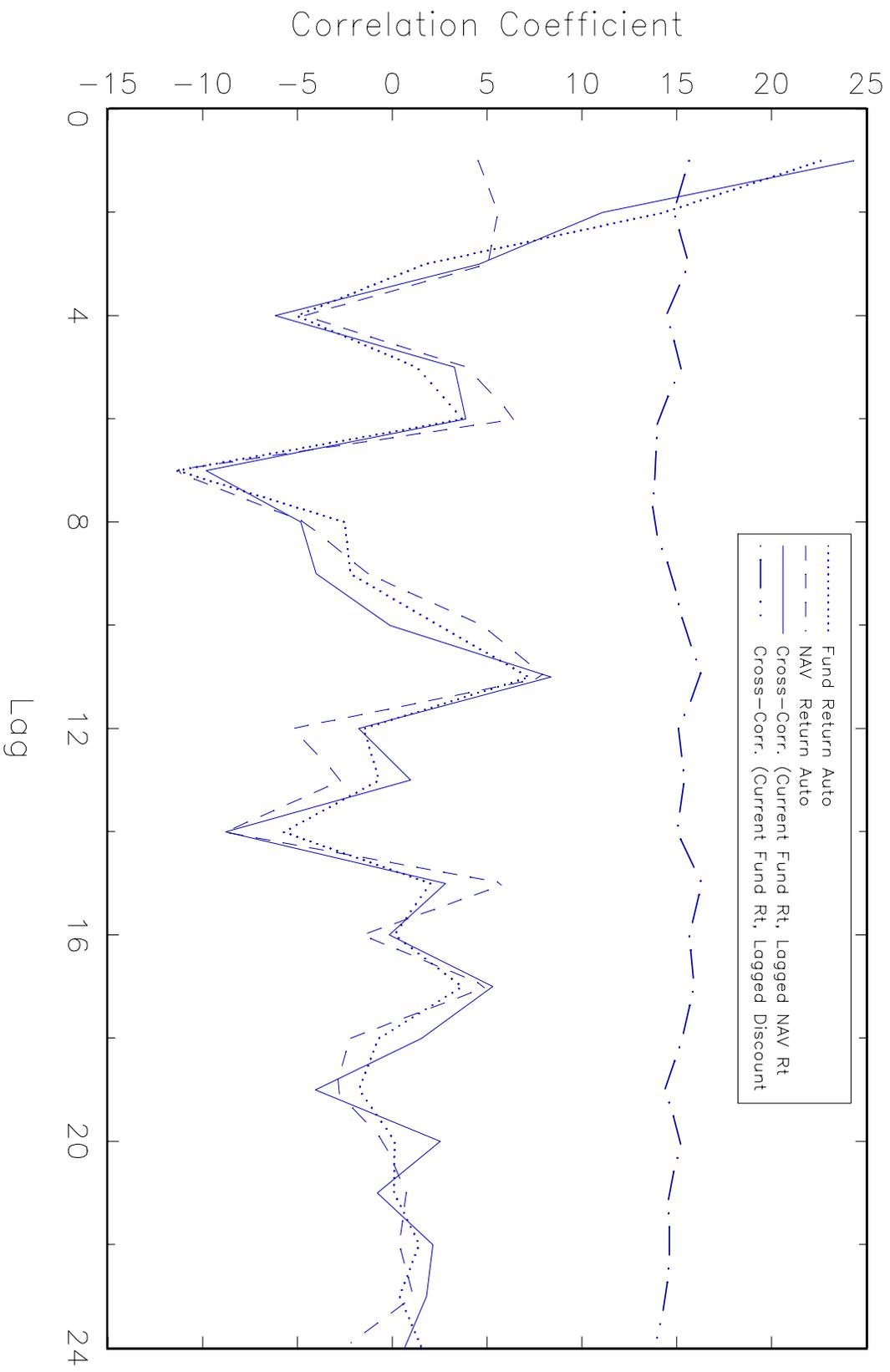


Figure 1. Correlation and Cross-correlation Coefficients at Different Lags [1993.1–2002.12]

Figure 2. Calibrated Correlation and Cross-correlation Coefficients at Different Lags

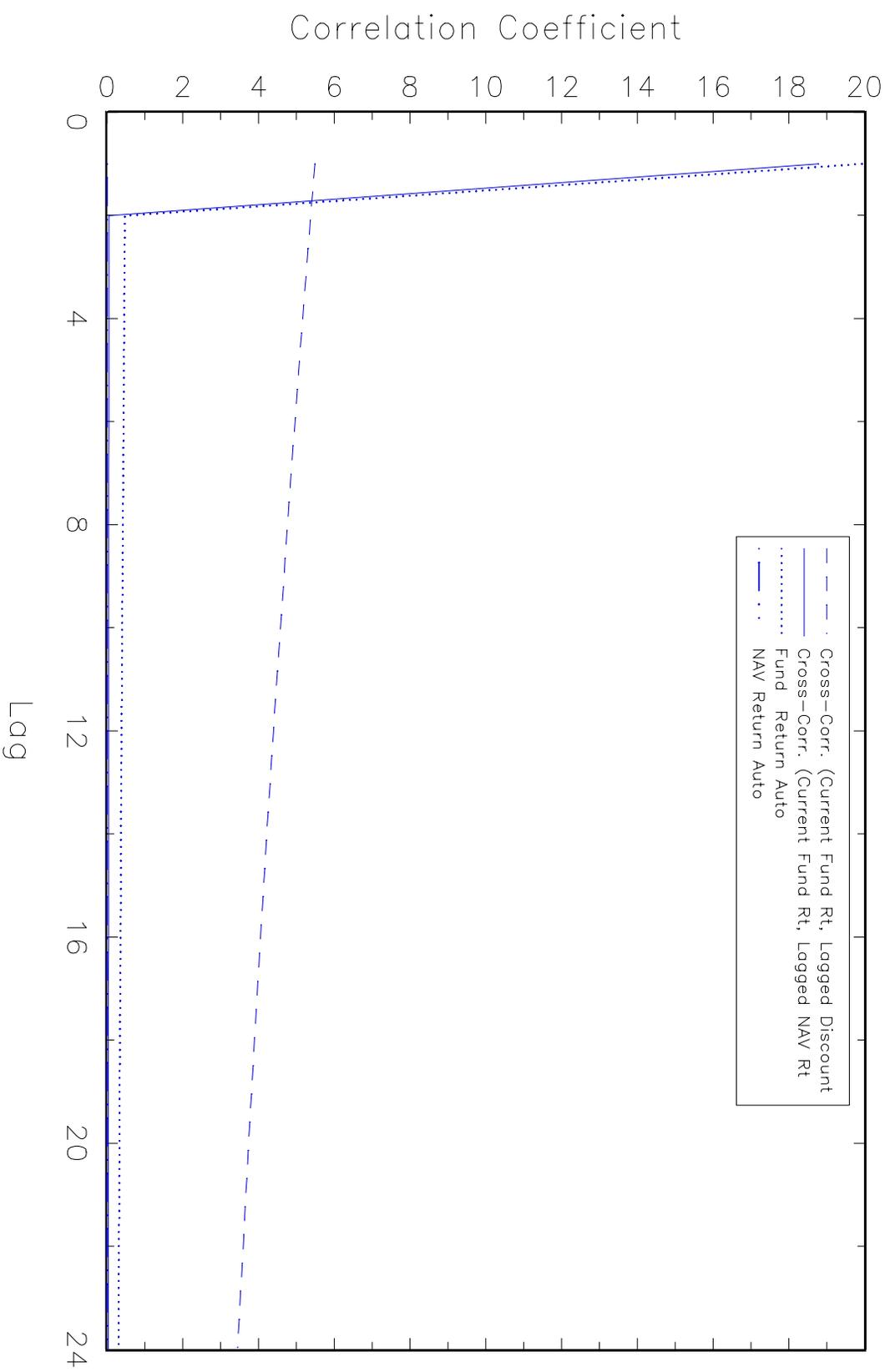
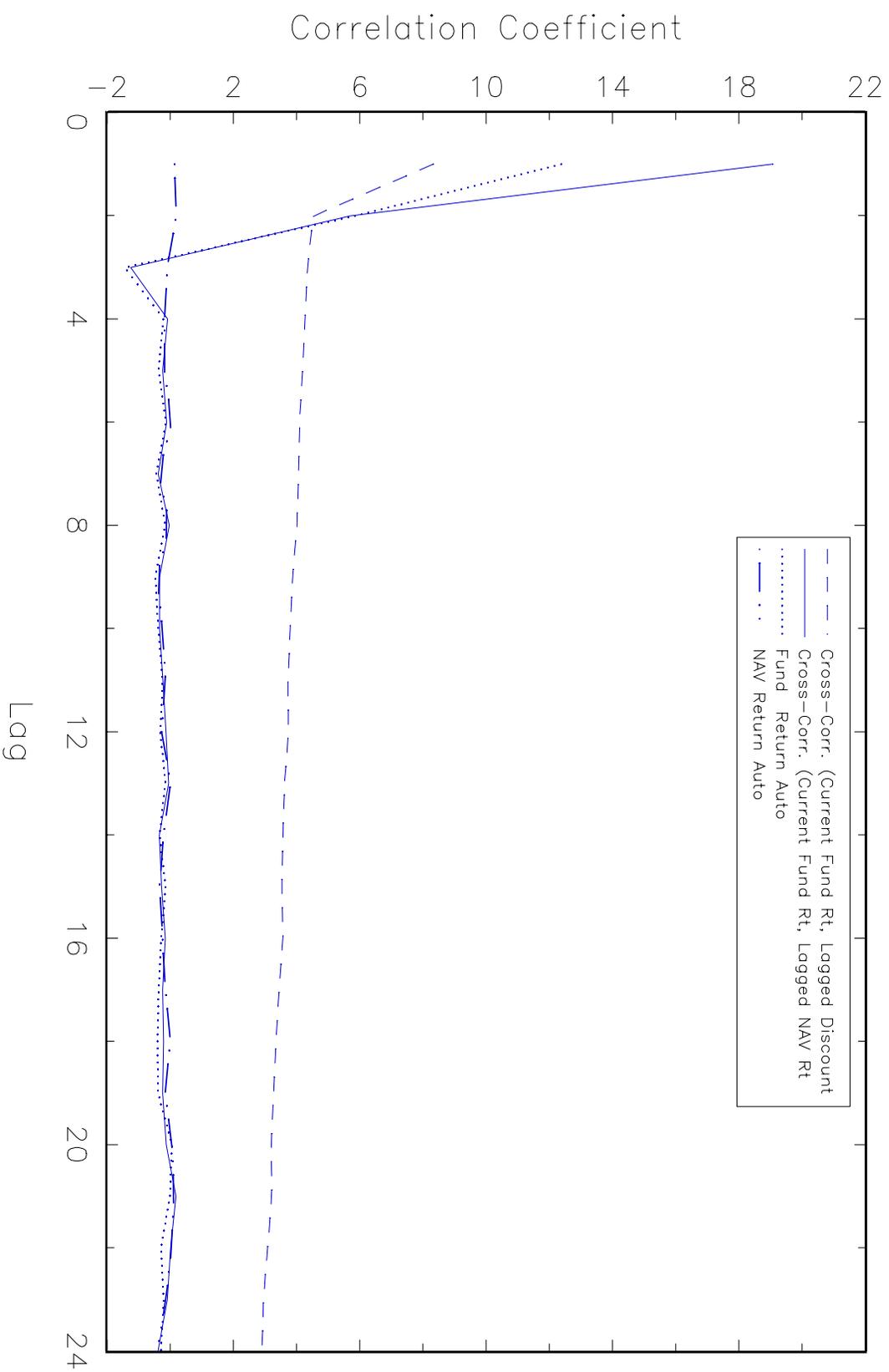


Figure 3. Simulated Correlation & Cross-correlation Coefficients at Different Lag  
[with Discounts Following an AR (2) Model]



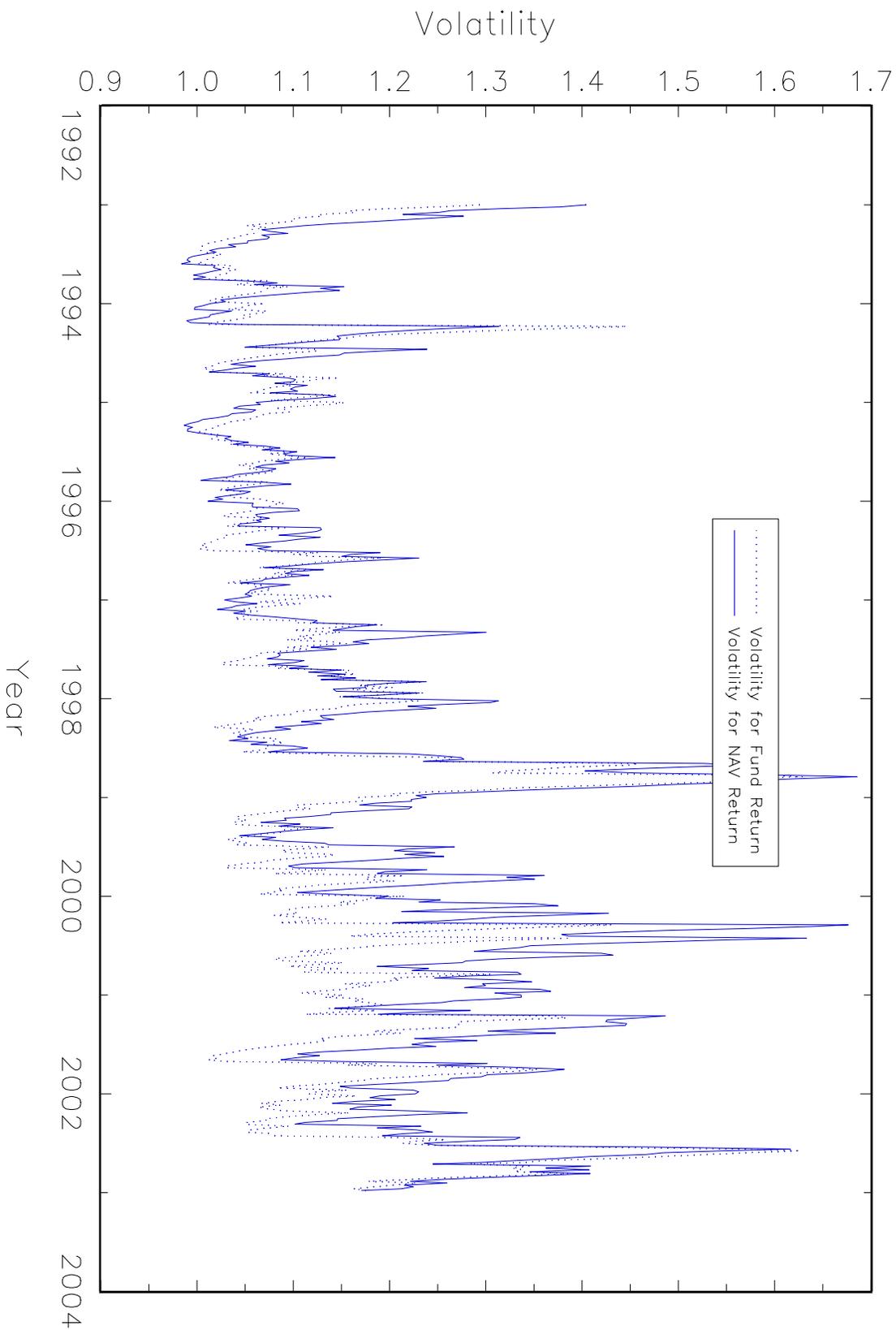


Figure 4. Conditional Volatility Using GARCH(1,1) Model [1993–2002]